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DISPLAY OF SAMPLED IMAGERY

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Abstract

Emphasis is often placed on the sample rate or spacing of sampled imagery, with less attention paid to how the output is constructed using the samples. The fidelity of a sampling process, however, depends on the reconstruction method as well as on the sample rate. This paper will discuss the benefits of display processing to improve image quality in sampled sensors. Topics will include: basic sampling theory, the application of Fourier Transforms to sampled imaging systems, the origin of phase artifacts in sampled imagery, and a discussion of the benefits of display processing when using more display pixels than sensor samples. Examples will be shown of images generated using different reconstruction techniques.

1.0 Introduction

It is widely recognized that a small sample interval is important in order to achieve good replication of sampled imagery. However, the fidelity of a sampling process is also affected by the method used to construct the output. In many applications one display pixel is used for each sensor sample. Further, newer systems might use flat panel displays with square, rectangular or round pixels which are unlikely to match the shape of the original image between the samples. Depending on display resolution, this mismatch might result in poor imagery or image artifacts.

A higher resolution display is required to improve the displayed image, but it may not be necessary to take additional sensor samples. Once a minimum sample rate

has been reached for a given sensor, improved display techniques can substitute for increased sampling and perhaps save system cost and complexity. The shape of the signal between the samples can be estimated or "interpolated" from the sample data; these interpolated values can then be used to make the displayed image a closer match to the original scene.

The benefits of display processing to improve image quality will be discussed. In Section 2.0, the process of sampling and reconstructing a signal is illustrated; the dependence of replicated signal fidelity on reconstruction technique is shown. In the next section, two expressions for the Fourier Transform of the reconstructed signal are derived. The first expression is used to clarify the reasons why a "system function" can not generally be associated with a sampled process. The second Fourier expression provides more insight into the signal reconstruction process and is the basis for most of what follows.

Section 4.0 of the paper provides an example of typical display processing and describes the associated problems: loss of high frequency components in the signal and sample phase artifacts. Alternate reconstruction methods are then discussed in Section 5.0 and examples given in Section 6.0 to illustrate the benefits.

2.0 Sampling and Reconstruction

In Figures 1 through 3, the function f(x) is sampled by finding the value at uniformly spaced intervals as indicated by the large asterisks (*). If (n) samples are taken with spacing (X), an approximation g(x) to f(x) can be reconstructed:

$$g(x) = \sum_{\text{all } n} f(nX)\cdot r(x-nX) \qquad \text{Equation 1}$$

where r(x) is a reconstruction function. The figures show three versions of g(x) using different r(x); in each case, r(x) is shown graphically as an insert. Figure 1 uses a rectangular function for reconstruction; bi-linear interpolation is used in Figure 2. In Figure 3, r(x) is the product of a sinc function times a Gaussian function. The selection of reconstruction function r(x) and sample interval (X) are fundamental to the fidelity with which g(x) approximates f(x). In each of the examples shown, g(x) equals f(x) at the sample points; the match between g(x) and f(x) improves from Figure 1 through Figure 3 because the different r(x) provide progressively better interpolation between sample points.

3.0 Fourier Transform of the Reconstructed Signal

This section provides background for the subsequent discussion on how the display affects image quality. The concept of reconstruction filter is described, and the reasons given why a Modulation Transfer Function (MTF) can not be defined for sampled systems.

If R(w) is the Fourier Transform of r(x) defined in Section 1.0, then $R(w) \cdot \exp^{-jwnX}$ is the transform of r(x-nX). Since the reconstructed signal is just a sum of offset reconstruction pulses r(x-nX), the Fourier Transform of the reconstructed signal g(x) in Equation 1 is

$$G(w) = \sum_{\text{all } n} f(nX) \cdot R(w) \cdot \exp^{-jwnX} \qquad \text{Equation 2}$$

Equation 2 helps to illustrate why a MTF or "system function" can not be assigned to a sampled process. An MTF exists if the input to the system can be shifted without changing the frequency content of the output; the output would shift coincident with the input, but the shape would not be altered. If the output exhibits this invariance under a shift of the input, the system is said to be constant parameter, and the output spectrum will be the product of the input spectrum and a system function.

In Equation 2, R(w) is not acting to weight or filter the input spectrum F(w); that is, G(w) does not equal $F(w) \cdot R(w)$. In the limit of (X) small and (n) large, the discrete transform:

$$F_{discrete}(w) = \sum_{\text{all } n} f(nX) \cdot \exp^{-jwnX}$$

does approximate the continuous transform F(w);[1] however, in most practical systems, the sample rate is limited and the attributes of a constant parameter system will be missing. That is, a sinusoidal input at frequency w_0 will not necessarily and solely result in a sinusoidal output at the same frequency with amplitude ratio $R(w_0)$. Further, the output spectrum G(w) is not uniquely determined by F(w) and R(w), but also depends on how (where) f(x) is sampled.[2]

An alternate and potentially more useful expression for the Fourier Transform of g(x) can be derived. By treating the output as having resulted from convolving r(x) with delta functions at the sample points, Equation 1 can be re-written:[3]

$$g(x) = \sum_{\text{all } n} \int_{-\infty}^{+\infty} f(x')\cdot\delta(x'-nX)\cdot r(x'-x)dx'$$

$$= \sum_{\text{all } n} \{f(x)\cdot\delta(x-nX)\} * r(x)$$

where (*) represents convolution. Since convolution in space results in multiplication in frequency, G(w) becomes:

$$G(w) = \sum_{\text{all } n} \{F(w)\cdot e^{-jwd} * \delta(w-nw_s)\}\cdot R(w)$$

$$w_s = 1/(2\pi X)$$

and r(x) is assumed to be symmetrical about zero (a sample point). For generality the zero of f(x) can offset by a distance (d). Then:

$$G(w) = \sum_{\text{all } n} F(w-nw_s)\cdot e^{-j(w-nw_s)d}\cdot R(w) \qquad \text{Equation 3}$$

Figure 4 is a notional plot of G(w). The output spectrum can be represented as F(w) replicated at multiples of w_s and then weighted by R(w). Note that the figure just shows the amplitude of the various spectra; the phase of the F(w) replicas varies due to the $e^{jnw_s d}$ multiplier. The shape of the output waveform (i.e., the inverse transform of the "spectrum of reconstructed signal" shown in the figure) will vary with sample phase.

Equation 3 shows why R(w) is sometimes called a "reconstruction filter." In this expression for G(w), R(w) is a multiplicative factor and has the appearance of a system function. This expression for the transform shows clearly what the desirable properties of R(w) should be: unit amplitude response in the frequency band of the original signal and a sharp drop in response at higher frequencies. Equation 3 also provides insight on the implications of using a less than ideal reconstruction function.

It should be noted that the sum of $F(w-nw_0)$ terms does not represent the frequency spectrum of a "sampled f(x)" but rather the transform of a series of delta functions. The delta functions do not have physical meaning outside their defining integral,[1,4] but we often engage in the convenient engineering fiction that the spectrum consisting of all $F(w-nw_0)$ terms is the transform of a series of infinitely bright, infinitesimally small spots at the sample points. It is these very small, very bright spots that we are "filtering" with the reconstruction. The actual samples are just the value of f(x) at spaced intervals; it is not physically sensible to associate a Fourier Transform with a list of numbers.

In order to achieve a good reproduction of the sampled signal, the sampling interval (X) must be small enough that the replicas of F(w) do not overlap significantly; that is, aliasing must be minimized. The reconstruction function r(x) serves two purposes. First, the function's transform should pass the desired frequencies F(w); if substantial aliasing does occur, however, it could be potentially beneficial to use R(w) to filter out the aliased portion of the spectrum. Second, the reconstruction should filter out the higher order replicas of F(w). Remnants of the high frequency replicas represent coherent changes to the image spectrum; they represent a degradation in the fidelity of the replicated signal. In practice, many of the image artifacts labeled as "aliasing" result in fact from the presence of the spurious higher frequencies and not from spectrum overlap; these artifacts can be corrected at the display.

4.0 Display of Sampled Imagery Without Display Processing

In many cases, the eye and normal display blur spots do not provide the ideal reconstruction described in Section 3.0; they do not provide a sharp differentiation between the desired original spectrum and the replicas at each sample frequency. A simplified model of a second generation thermal imager will serve as an example. In Figure 5, the "original spectrum" represents the transform of the analog output from an F/3 narrow field of view sensor using the Army standard focal plane. In this case, the optical blur is somewhat larger than the detector and sampling rate is two samples per dwell. The sampled signal would actually be the input scene convolved with the sensor MTF. For purposes of this example, we will use the MTF itself; consider the scene to be a point source or highly random. The MTF for the display and eye are based on a 6 inch by 8 inch, flat panel display with 50% pixel fill factor and a 28 inch eye distance.

This example sensor is well sampled, perhaps over-sampled; adjacent replicas of the input spectrum do not overlap. As shown in Figure 5, this eye/display combination causes only a slight drop in the high frequency spatial response of the sensor system; i.e., the high end of the original spectrum is not attenuated. However, the eye/display combination does not completely "filter out" the adjacent replica of the input spectrum. As illustrated in the figure, the image spectrum has significant components beyond half the sample frequency. This high frequency signal is not "noise;" it represents a change in shape or intensity pattern of the image. Although significant aliasing is not present, sample phase artifacts would still be visible.

If the sampling artifacts prove to be bothersome, one corrective action is to increase the sample rate and improve display resolution. The increased sample rate

would further separate the high frequency replicas from the original spectrum; the eye/display filtering would then be more effective, although the display MTF would increase due to the smaller pixel size. Figure 6 illustrates what happens when the sample rate is doubled; the replicate spectra are more widely spaced, with the result that the eye provides better filtering of the unwanted high frequencies. The spectrum of the output is a closer match to the input spectrum.

Improved image quality at the display can be supported without increasing sensor sample rate. Since significant aliasing is not present even at the lower sample rate shown in Figure 5, values of the input signal at the mid-points between samples can be interpolated from that data. That is, we find an r(x) with transform F(w) that does not degrade the desired spectrum but will filter out the adjacent replicate spectra; this r(x) is convolved (mathematically, in a computer or processor) with the input data to generate an estimate of the value of the input signal at points intermediate to the sample points; the expanded set of samples, original data plus interpolated values, drive the higher resolution display.

If sensor imagery is just sufficiently sampled to avoid significant aliasing, but not excessively sampled, then the replicated spectra above and below the base-band (original) spectrum will be closely spaced to the original but not overlapping. In order to discriminate the original spectrum from the replicates, we would like an R(w) with a flat "passband" to frequency $w_s/2$ and a sharp cutoff beyond.

5.0 Reconstruction Functions

In theory a perfect r(x) exists, but perfection can not be attained in a realizable system. However, very good reconstruction functions can be easily implemented. In this section, the Sampling Theorem will be described and its practical limitations discussed; examples will then be given of practical extensions of the Sampling Theorem.

Referring to Figure 4 and Equation 2, if f(x) is band-limited to $w_s/2$ so that the replicas of F(w) do not overlap, and if R(w) is a rectangular function centered at zero and with full width w_s, then G(w) will equal F(w) and f(x) will be perfectly replicated. Since the Fourier Transform of a rectangular function is a sample function [sin(x)/(x)], Equation 1 becomes:

$$g(x) = \sum_{\text{all } n} f(nX)\cdot\sin(\pi x/X - n\pi)/(\pi x/X - n\pi) \qquad \text{Equation 4}$$

The Sampling Theorem states that, for a signal f(x) for which the Fourier Transform has no components above $w_s/2$ inclusive, the function can be entirely reconstructed by the above series of sample functions. That is, a band-limited function can be uniquely determined from its values at a sequence of equidistant points, $1/w_s$ apart.[2] The Sampling Theorem provides the capability to find the value of f(x) at points intermediate to the sample points.

The Sampling Theorem has practical limitations in that realizable signals can not be ideally sampled. A signal can not be both band-limited and of limited extent;[5] an ideal reconstruction would require either infinite sample rate or gathering an infinitude of samples. Although the work of Landau and Pollak [2,5] shows that the sample function expansion is not theoretically the best choice for realizable signals, in practice the Sampling Theorem does hold in an approximate sense and is a useful guide.

The Sampling Theorem is awkward to apply because the sample function falls off slowly. In Equation 4, if we are interested in approximating g(x) at a point, the contribution to the approximation from a distant sample falls off inversely with the distance to the sample. In an imaging sensor, samples distant by more than a spread function diameter are not contributing real information content to the approximation. However, convolution with the sample function can not be arbitrarily truncated since contributions from distance samples tend to cancel the contributions from adjacent distant samples. Truncating a sinc wave reconstruction function results in a new reconstruction function with a transform which is the convolution of a rectangular function with a sinc wave. The new R(w) has very poor reconstruction properties.

Reconstruction functions which have a width of only a few samples and retain the desired frequency characteristics can be generated by applying a smooth window to the sample function. The reconstruction function used in Figure 3 is a sample function multiplied by a Gaussian which falls to about 3% amplitude at the third sample and the function is truncated at the third sample. A similar technique can be used to generate reconstruction functions of different widths; a wider function has a steeper cutoff at the desired band-limit.

Figure 7 shows the reconstruction filters obtained when using functions with width of four, six and eight samples. The calculation of each interpolation point would involve a series with four, six or eight terms, respectively. Also shown in the figure is the transform of the triangular wave used in Figure 2. Using the triangular function for reconstruction provides a two point interpolation. The desirable qualities of a reconstruction function, unit amplitude in the passband and a sharp cutoff, are enhanced by using more points in the reconstruction.

6.0 Examples

In this section, examples will be given to illustrate that image quality is enhanced by using the reconstruction techniques described in Section 5.0. It is quite often possible to significantly enhance displayed image quality without taking additional sensor samples.

The baseline images, referred to below and in the figures as "original," represent good quality, black and white video. These images are obtained using a digital frame grabber and a high quality, Charge Coupled Device camera. The Cohu camera is a frame transfer device with 760 horizontal by 480 vertical pixels. The camera output is digitized with a Data Translation video frame grabber taking 512 by 512 samples with 8 bit accuracy. The digitizer electronics limits the video bandwidth to 4.5 MHz.

The original image at the top left of Figure 8 shows a framed picture of an AH-1 Cobra Helicopter hanging on the laboratory wall. At top right in the figure, the helicopter image is zoomed eight times by pixel replication. The bottom, right picture in Figure 8 is generated by zooming the helicopter image eight times using bi-linear interpolation. Modified Sample Theorem interpolation was used to process the bottom, left-hand picture. Sample Theorem reconstruction clearly provides the best image quality.

In Figure 9, the original image is down-sampled to 256 by 256 pixels. Three out of four of the original samples are discarded in order to represent a sensor which takes one forth the samples. The top right picture shows the result of replicating the down-sampled image sixteen times. The bottom, right picture results from bi-linear interpolation and the bottom, left picture from Sample Theorem reconstruction. The images at the bottom of Figure 9 look like Cobra Helicopters even though the reconstruction uses only a quarter of the sensor samples compared to Figure 8. If the top, right picture in Figure 9 is used as the basis of comparison, the improved image quality represented by the bottom, left picture in Figure 9 is obtained solely through the use of display reconstruction techniques, whereas four times as many sensor samples are required to achieve the improved image quality represented by the pictures in Figure 8.

The original image of a face is shown at the top, left-hand side in Figure 10. To the right in the figure, this face is down-sampled to 256 by 256 pixels. The bottom, left picture results from Sample Theorem reconstruction. Only minor differences can be discerned.

The top, right image in Figure 11 is obtained by four pixel replications of the original face shown at the top, left of Figure 10. The top, left picture in Figure 11 results from eight pixel replications of the down-sampled image shown at the top, right of Figure 10. The bottom image in Figure 11 is a Sample Theorem reconstruction from the down-sampled data. The picture constructed from fewer samples using good reconstruction techniques is a close match to the pixel replicated image which is generated from four times the number of samples.

7.0 Conclusion

In many practical applications, poor image quality or sample artifacts result from poor signal reconstruction rather than from inadequate sampling. Good replication of a sampled image requires that sample spacing be small enough that aliasing does not occur. However, once a minimum sample rate has been reached for a given sensor, improved display techniques can substitute for increased sampling and perhaps save system cost and complexity.

The examples illustrate that image quality can be improved by display processing. Further, near ideal reconstruction functions exist which are only a few samples wide. Improved image reconstruction by interpolating the input data is a practical alternative to taking additional sensor samples.

8.0 References

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4. Chisholm, J.S.R., and Morris, R.M., Mathematical Methods in Physics," Saunders, Philadelphia, 1965, Chapter 18.

5. Wozencraft, J.M., and Jacobs, I.M., "Principles of Communication Engineering," John Wiley & Sons, New York, 1965, page 682 and Appendix 5B.

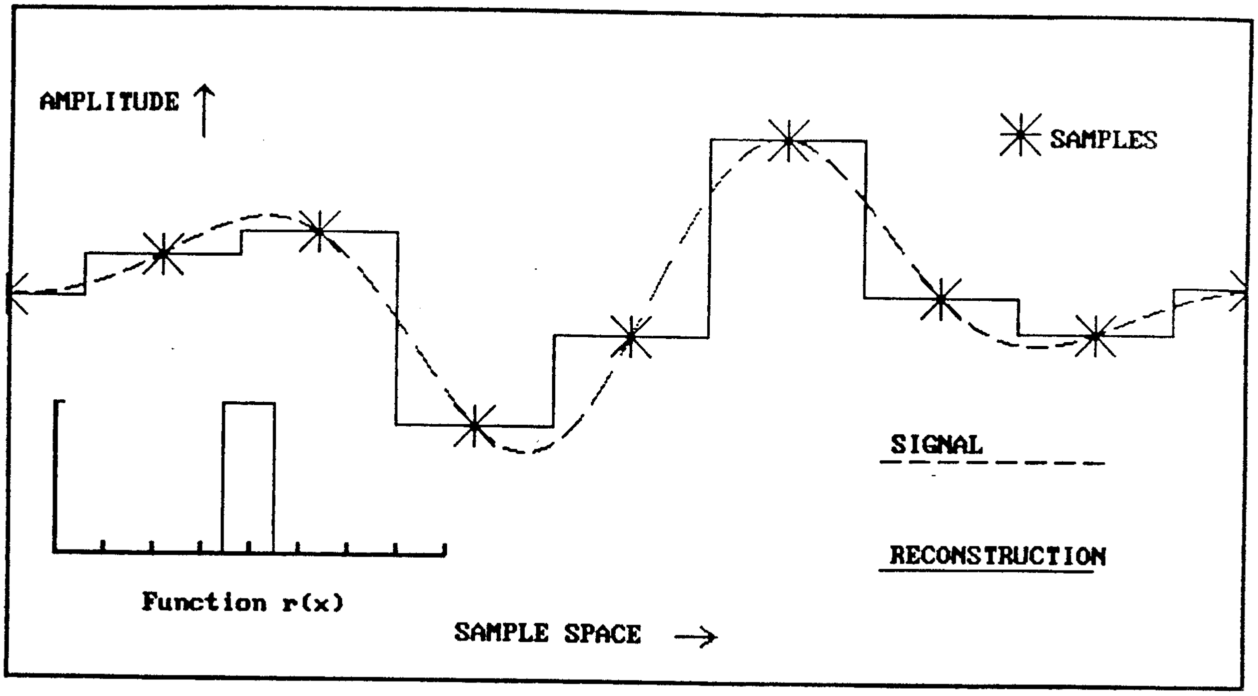


Figure 1. Reconstruction of a Signal using a Rectangular Function to Replicate the Samples.

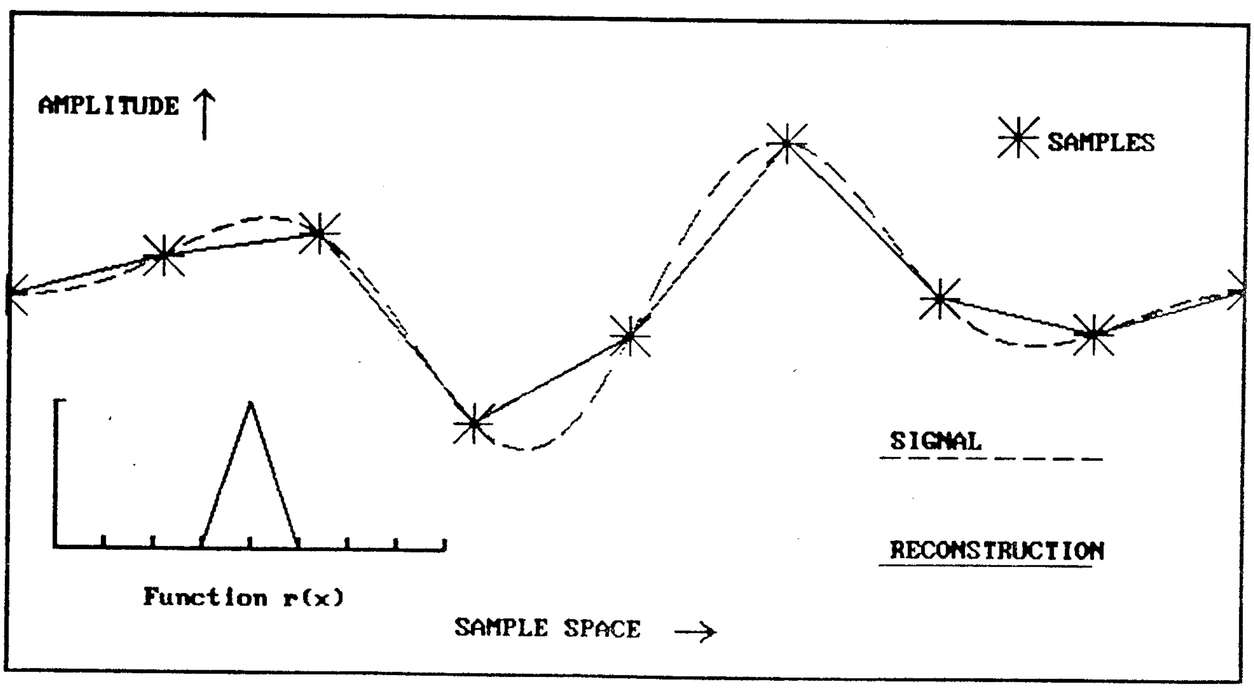


Figure 2. Reconstruction of a Signal using a Triangular Function to Bi-linear Interpolate.

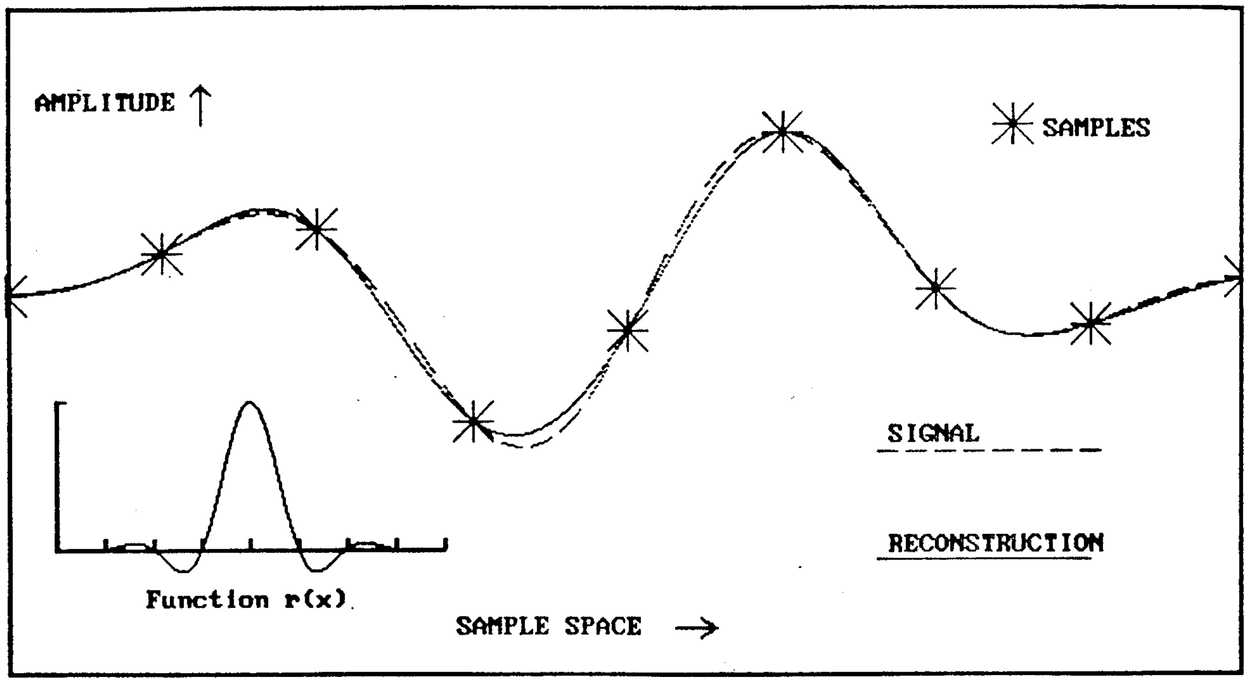


Figure 3. Reconstruction of a Signal using a Gaussian Windowed Sinc Function

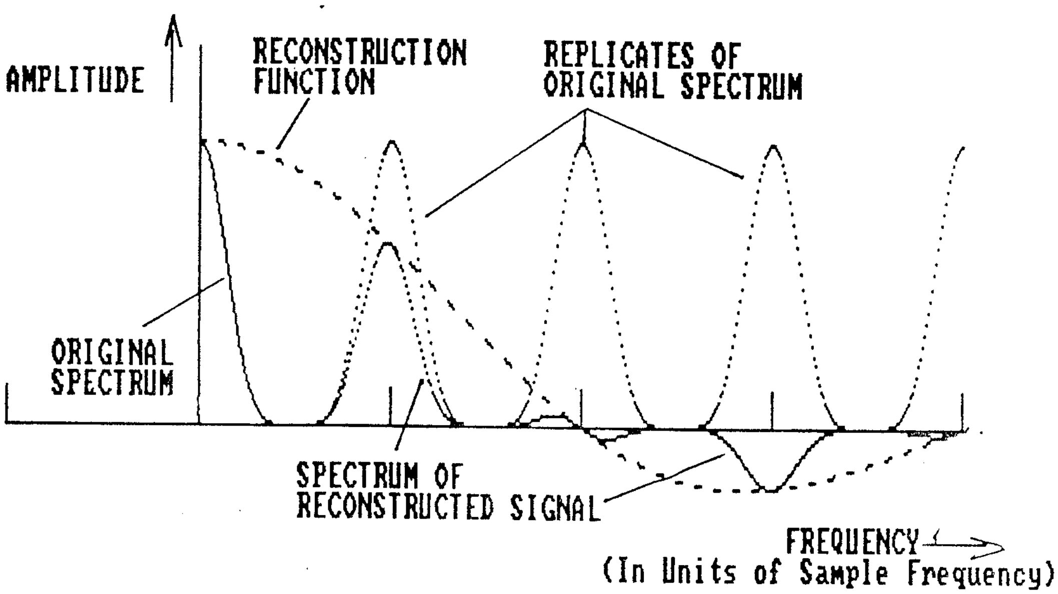


Figure 4. Notional Plot of F(w) Replicated at Sample Frequencies and "Filtered" by R(w).

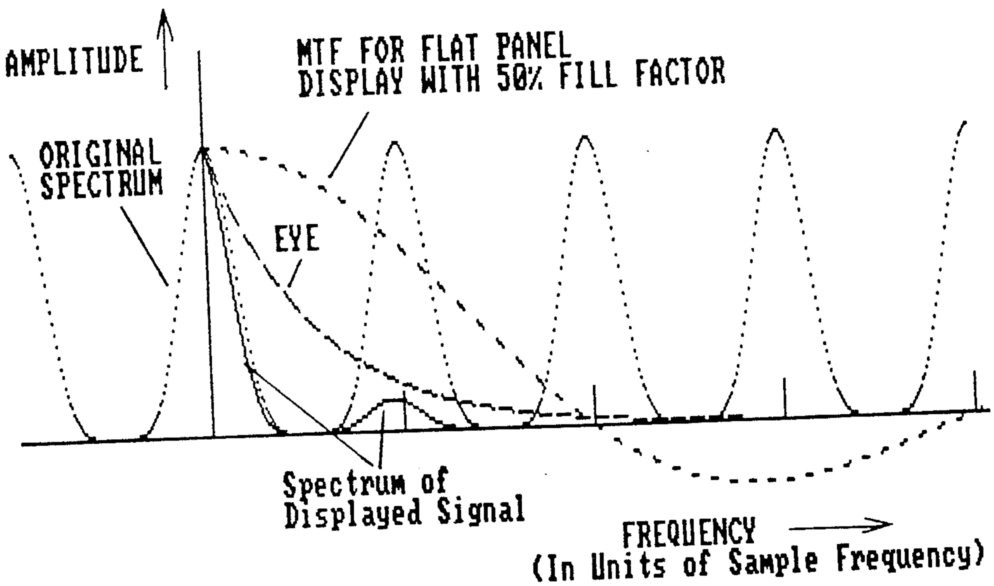


Figure 5. Spectra Associated with the Sampled Output of the Hypothesized Thermal Imaging system.

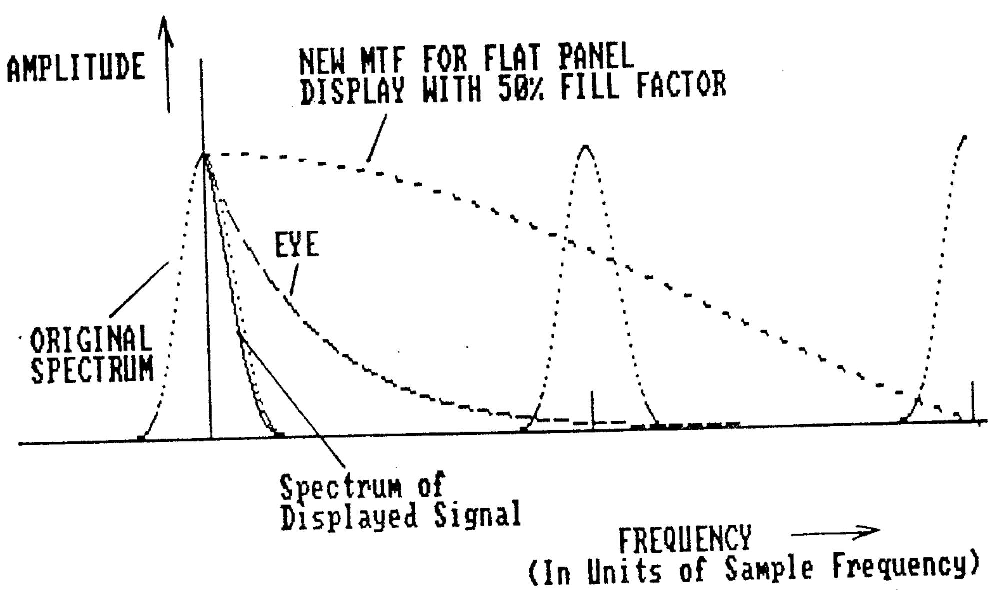


Figure 6. Spectra Associated with Doubling the Sample Rate in Figure 5 Above.

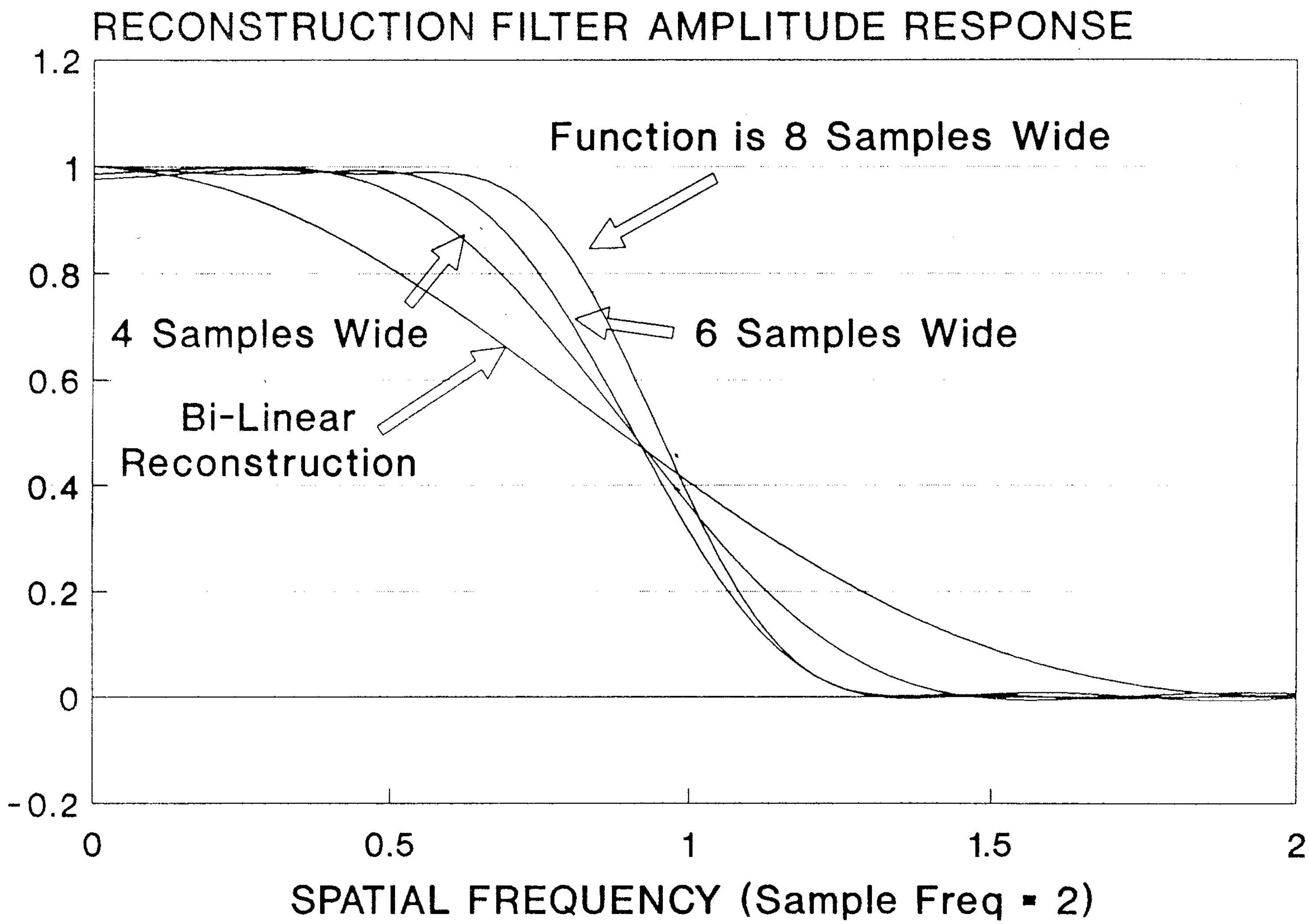
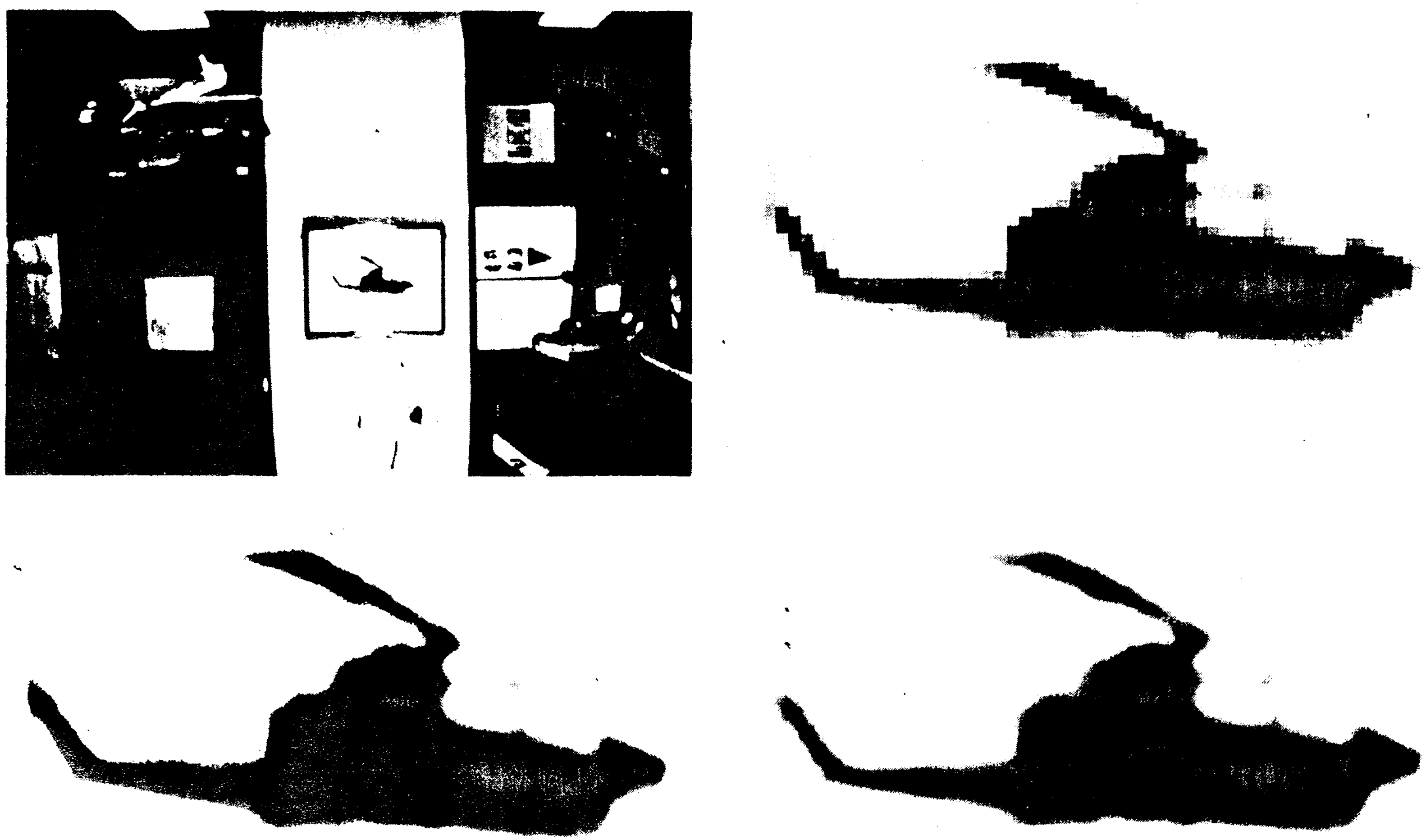


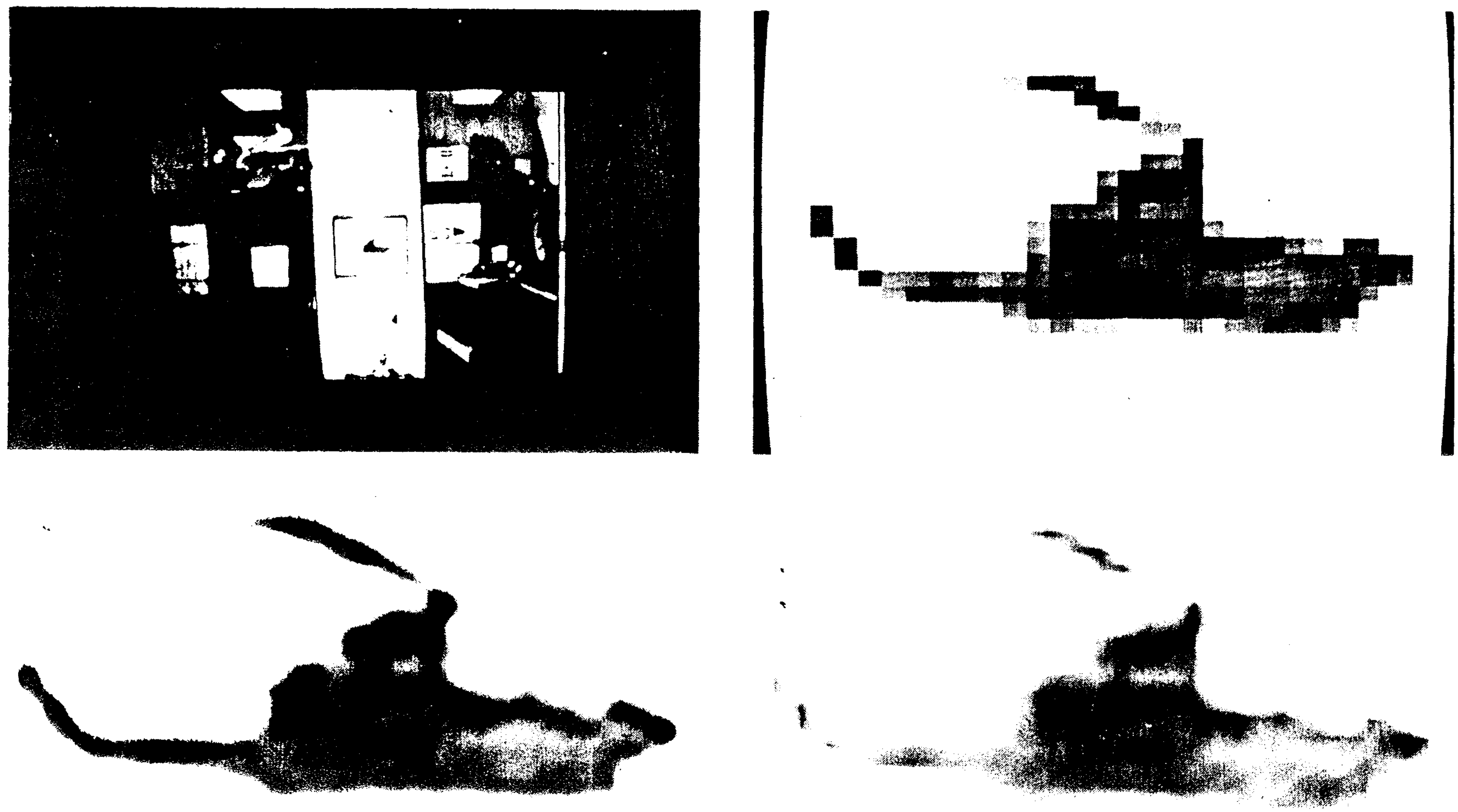
Figure 7. Example R(w) for Functions of Limited Width.



TOP LEFT: ORIGINAL
BOTTOM LEFT: SAMPLE THEOREM

TOP RIGHT: PIXEL REPLICATE
BOTTOM RIGHT: BI-LINEAR

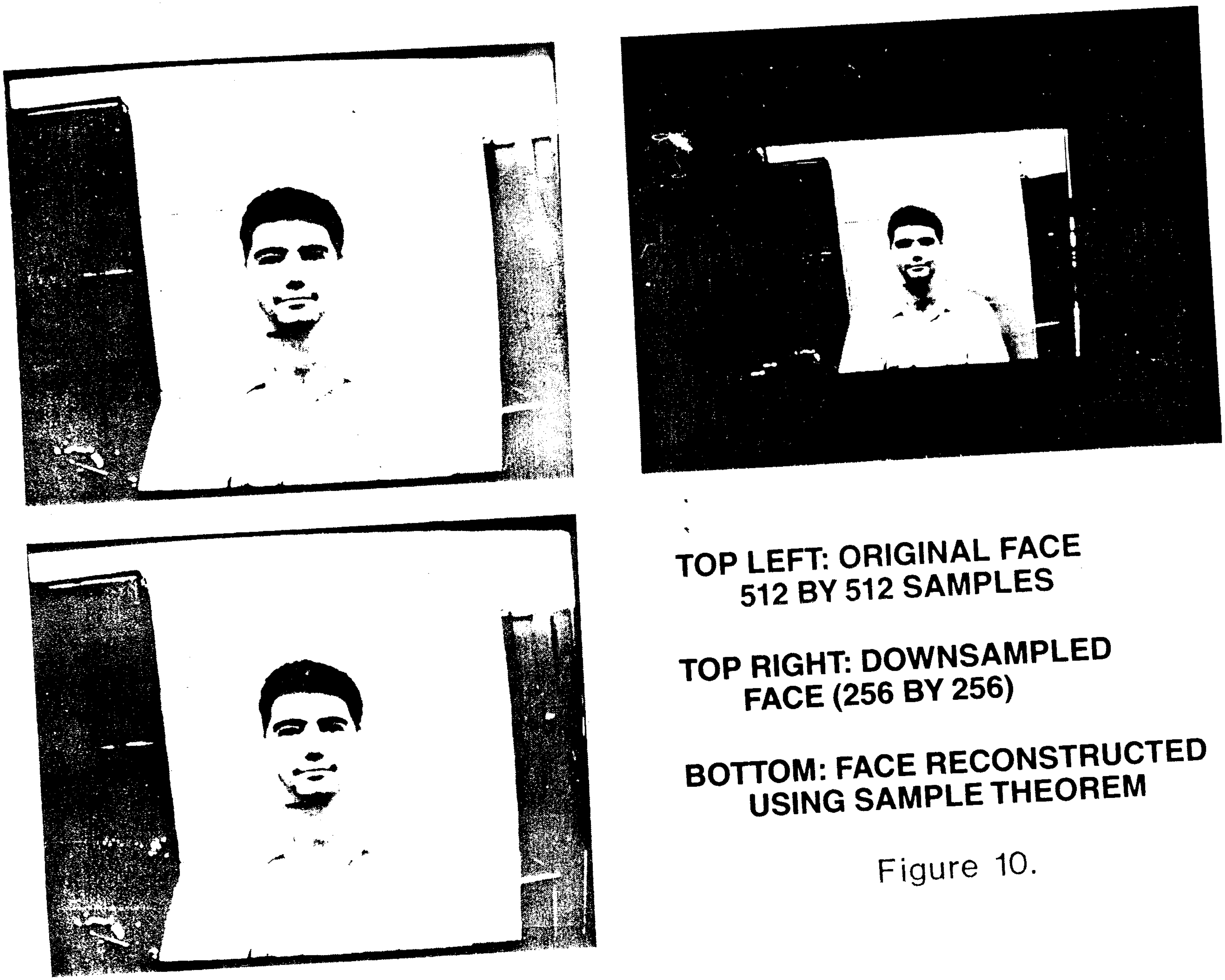
Figure 8. Picture of Cobra Helicopter Zoomed 8 times
Using the r(x) Shown in Figures 1-3.



TOP LEFT: DOWN-SAMPLED IMAGE
BOTTOM LEFT: SAMPLE THEOREM

TOP RIGHT: PIXEL REPLICATE
BOTTOM RIGHT: BI-LINEAR

Figure 9. Down-sampled Cobra Picture Zoomed 16 Times by r(x) Shown in Figures 1-3.

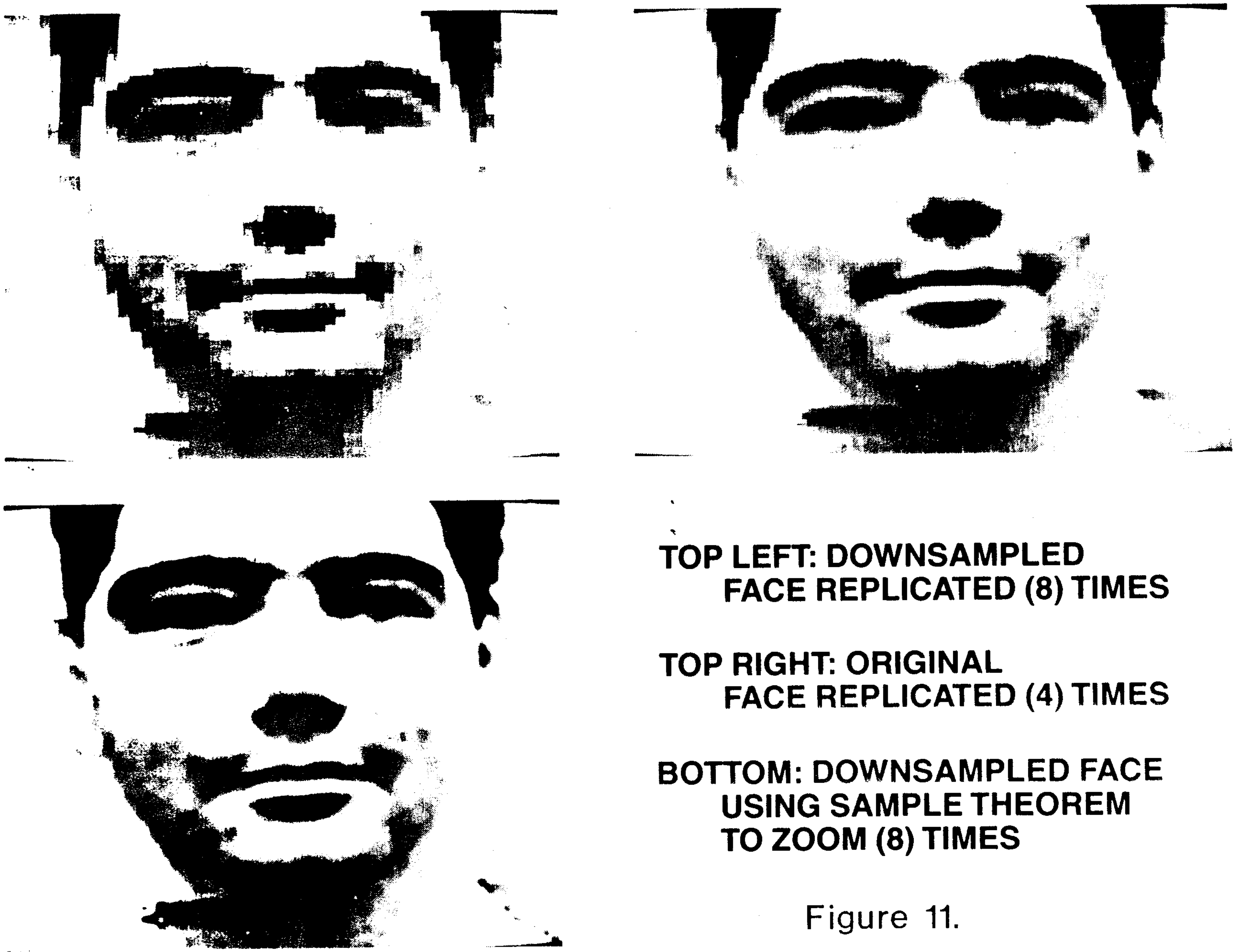


TOP LEFT: ORIGINAL FACE
512 BY 512 SAMPLES

TOP RIGHT: DOWNSAMPLED
FACE (256 BY 256)

BOTTOM: FACE RECONSTRUCTED
USING SAMPLE THEOREM

Figure 10.



TOP LEFT: DOWNSAMPLED FACE REPLICATED (8) TIMES

TOP RIGHT: ORIGINAL FACE REPLICATED (4) TIMES

BOTTOM: DOWNSAMPLED FACE USING SAMPLE THEOREM TO ZOOM (8) TIMES

Figure 11.